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$t\bar{t}$ cross section near the production threshold in NNLO of NRQCD

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Abstract

The cross section near the $t\bar{t}$ production threshold in e^+e^- annihilation and $\gamma\gamma$ collisions is discussed. The NNLO results within NRQCD are briefly overviewed.

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1. Introduction.

The hadron production near heavy quark threshold will be thoroughly studied experimentally at future accelerators, e.g. [1]. The motion of the heavy quark-antiquark pair near the production threshold is nonrelativistic to high accuracy that justifies the use of the nonrelativistic quantum mechanics as a proper theoretical framework for describing such a system [2, 3, 4]. Being much simpler than the comprehensive relativistic treatment of the bound state problem with Bethe-Salpeter amplitude [5], this approach allows one to take into account exactly such an essential feature of the near-threshold dynamics as Coulomb interaction [6, 7]. For unstable heavy quarks with a large decay width it is possible to compute the cross section near threshold point-wise in energy because the large decay width serves as an infrared cutoff and suppresses the long distance effects of strong interaction [8].

2. $t\bar{t}$-system near threshold

The $t\bar{t}$-pair near the production threshold is just a system that satisfies the requirement of being nonrelativistic. Therefore the description of $t\bar{t}$-system near the production threshold $\sqrt{s} \approx 2m_t$ ($\sqrt{s}$ is a total energy of the pair, m_t is the top quark mass) is quite precise within NRQCD. Reasons for this accuracy are related to the large mass of the top:

- The top quark is very heavy $m_t = 175$ GeV [9] and there is an energy region of about $8 - 10$ GeV near the threshold where the nonrelativistic approximation for the kinematics is very precise. The velocity is small (for E of interest in the range $|E| \sim 2 \div 5$ GeV)

$$v = \sqrt{1 - \frac{4m_t^2}{s}} = \sqrt{1 - \frac{4m_t^2}{(2m_t + E)^2}} \simeq \sqrt{\frac{E}{m_t}} \approx 0.10 \div 0.15 \ll 1\,. \qquad (1)$$

  Relativistic effects are small and can be taken into account perturbatively in v (even in v^2).

- the strong coupling constant at the high energy scale is small $\alpha_s(m_t) \approx 0.1$ that makes the mapping of QCD onto the low energy effective theory (NRQCD), which is

perturbative in $\alpha_s(m_t)$, numerically precise.

- The decay width of top quark is large, $\Gamma_t = 1.43$ GeV; infrared (small momenta) region is suppressed and PT calculation for the cross section near the threshold is reliable even point-wise in energy.

The above properties could make the $t\bar{t}$ system near threshold flat and not interesting for physical study, however, there is a feature of quark-antiquark interaction that becomes dominant in this kinematical regime and brings a nontrivial structure into the dynamics. Because $\alpha_s \sim v$ and the ratio α_s/v is not small, the Coulomb interaction is enhanced. The ordinary perturbation theory for the cross section (with free quarks as the lowest order approximation) breaks down and all terms of the order $(\alpha_s/v)^n$ should be summed. The expansion for a generic observable $f(E)$ in this kinematical region has the form

$$f(E) = f_0(\alpha_s/v) + \alpha_s f_1(\alpha_s/v) + \alpha_s^2 f_2(\alpha_s/v) + \dots \tag{2}$$

where $f_i(\alpha_s/v)$ are some (not polynomial) functions of the ratio α_s/v, $f_0(\alpha_s/v)$ is a result of the pure Coulomb approximation (or a kind of its improvement). The expansion in α_s in eq. (2) takes into account the perturbative QCD corrections to the parameters of NRQCD and relativistic corrections (in the regime $v \sim \alpha_s$). One can study the $t\bar{t}$ system near threshold in the processes $e^+e^- \to t\bar{t}$ [7, 10] and $\gamma\gamma \to t\bar{t}$ [11, 12]. These processes have the following features:

- $e^+e^- \to t\bar{t}$: the production vertex is local (the electromagnetic current in case of photon and/or weak current in case of Z-boson), the basic observable is a production cross section which is saturated by S-wave (for the vector current), NNLO analysis is available.

- $\gamma\gamma \to t\bar{t}$: the production vertex is nonlocal (T-product of two electromagnetic currents), both S- and P-waves can be studied for different helicity photons, the number

of observables is larger (cross sections σ_S, σ_P, S-P interference). Because of nonlocality of the production vertex the high energy coefficient (necessary for mapping the QCD quantities to NRQCD ones) is more difficult to obtain. Some calculations were done in NLO [13, 14] and the full analysis of the cross section is available in NLO of NRQCD only [15]. The low energy part of the process can be studied in NNLO without a strict normalization to full QCD (see [16] for relativistic corrections).

3. Theoretical description.

I shall discuss only $e^+e^- \to t\bar{t}$ process mediated by the photon because NNLO analysis, which contains the most interesting features, is possible. The basic quantity is the vacuum polarization function

$$\Pi(E) = i \int \langle T j_{em}(x) j_{em}(0) \rangle e^{iqx} dx, \quad q^2 = (2m_t + E)^2 \,. \tag{3}$$

Near the threshold (for small energy E) NRQCD is used. The cross section is saturated with S-wave scattering. In this approximation the polarization function near the threshold to the NNLO accuracy in NRQCD is given by

$$\Pi(E) = \frac{2\pi}{m_t^2} C_h(\alpha_s) C_{\mathcal{O}}(E/m_t) G(E;0,0) \,. \tag{4}$$

The pole mass definition is used for m_t (e.g. [17]), α_s is the strong coupling constant. The choice of normalization points for coupling constants entering different parts of the theoretical expression (4) will be discussed later. $C_h(\alpha_s)$ is the high energy coefficient which has been known in the NLO since long ago [18, 19, 20] (before the explicit formulation of NRQCD). $G(E;0,0)$ is the nonrelativistic GF, $E = \sqrt{s} - 2m_t$. The leading term of the cross section in the effective theory representation is given by a correlator of currents with dimensionality 3 in mass units. The quantity $C_{\mathcal{O}}(E/m_t)$ describes the contributions of higher dimension operators within the effective theory approach. These contributions have, in general, a different structure than the leading term. However, to the NNLO of NRQCD the contribution of higher dimension operators can be written as a total factor $C_{\mathcal{O}}(E/m_t)$

for the leading order GF, $C_{\mathcal{O}}(E/m_t) = 1 - 4E/3m_t$. The polarization function near the threshold (4) contains expansions in small parameters α_s and/or v, cf. eq. (2). The leading order approximation of the low energy part is the exact Coulomb solution for the Green's function. The more detailed description of the ingredients of the representation in eq. (4) is:

- the nonrelativistic Green's function $G(E; x, x')$ is given by $G(E) = (H - E)^{-1}$ where

$$H = \frac{p^2}{m_t} + V(r) \tag{5}$$

is the nonrelativistic Hamiltonian describing dynamics of the $t\bar{t}$-pair near the threshold. The most complicated part of Hamiltonian (5) to find is the heavy quark static potential $\tilde{V}_{pot}(q)$ entering into the potential $V(r)$. The static potential $\tilde{V}_{pot}(q)$ is computed in perturbation theory

$$\tilde{V}_{pot}(q) = -C_F \frac{\alpha_s}{q^2} \left(1 + \alpha_s(b_1 \ln \mu/q + a_1) + \alpha_s^2(b_1^2 \ln^2 \mu/q + b_2 \ln \mu/q + a_2) + \ldots\right) . \tag{6}$$

Here $b_1 = 2\beta_0$, $b_2 = 2(\beta_1 + 2\beta_0 a_1)$,

$$(4\pi)\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f = \frac{23}{3} \tag{7}$$

is the first coefficient of the β-function,

$$(4\pi)^2 \beta_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F n_f - 4 C_F T_F n_f = \frac{116}{3} \tag{8}$$

is the second coefficient of the β-function. The static potential can be written in the form

$$\tilde{V}_{pot}(q) = -C_F \frac{\alpha_V(q)}{q^2} \tag{9}$$

that gives a definition of the effective charge α_V related to the $\overline{\rm MS}$-scheme coupling constant

$$\alpha_V(\mu) = \alpha_s(\mu)(1 + a_1 \alpha_s(\mu) + a_2 \alpha_s(\mu)^2) . \tag{10}$$

Coefficients $a_{1,2}$ are known. The numerical value for a_1 reads [21]

$$a_1 = \frac{1}{4\pi} \left(\frac{31}{9} C_A - \frac{20}{9} T_F n_f\right) = \frac{1}{4\pi} \left(\frac{43}{9}\right) .$$

The coefficient a_2 is given by [22, 23]

$$(4\pi)^2 a_2 = \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\zeta(3)\right) C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta(3)\right) C_A T_F n_f$$
$$- \left(\frac{55}{3} - 16\zeta(3)\right) C_F T_F n_f + \left(\frac{20}{9} T_F n_f\right)^2 ,$$

or in a more concise form,

$$a_2 = \frac{1}{(4\pi)^2}\left(\frac{7217}{162} + 36\pi^2 - \frac{9\pi^4}{4} - \frac{62\zeta(3)}{3}\right) = \frac{1}{(4\pi)^2} 155.842...$$

In QCD we have $C_A = 3$, $C_F = 4/3$, $T_F = 1/2$, and $n_f = 5$ for the energy region of the $t\bar{t}$-pair production. The effective coupling α_V is nothing but a running coupling constant in some special subtraction scheme. The coefficient a_2 allows one to find the effective β-function β_V for the evolution of the coupling α_V in the NNLO. Therefore the effective coupling constant for the static potential is now fully determined at the NNLO.

- High energy coefficient $C_h(\alpha_s)$ is given by the expression

$$C_h(\alpha) = 1 - 4\frac{\alpha_s}{\pi} + C_F\left(\frac{\alpha_s}{\pi}\right)^2 \left(-\pi^2\left(\frac{2C_F}{3} + C_A\right)\ln\frac{\mu_f}{m_t} + c_2\right) . \qquad (11)$$

In NNLO, there appears a new term proportional to the logarithm of the factorization parameter μ_f that separates long and short distances (or large and small momenta) within the effective theory approach. The finite (μ_f independent) coefficient c_2 is known [24, 25]

$$c_2 = \left(\frac{39}{4} - \zeta(3) + \frac{4\pi^2}{3}\ln 2 - \frac{35\pi^2}{18}\right) C_F - \left(\frac{151}{36} + \frac{13}{2}\zeta(3) + \frac{8\pi^2}{3}\ln 2 - \frac{179\pi^2}{72}\right) C_A$$
$$+ \left(\frac{44}{9} - \frac{4\pi^2}{9} + \frac{11}{9} n_f\right) T_F .$$

The coefficient of NLO in eq. (11) is μ_f independent – the factorization procedure (a separation of scales) is insensitive to the border. An explicit dependence of high

and low energy quantities on the factorization scale μ_f is a general feature of effective theories which are valid only for a given region of energy. A physical quantity, which is given by a proper combination of results obtained in different energy regions within respective effective theories, is factorization scale independent e.g. [26, 27]. In NRQCD this feature reveals itself in μ_f dependence of Green's function and of the high energy coefficient C_h. To see how this dependence emerges one can consider the contribution of a generic NNLO (three-loop) diagram into the cross section. In full QCD one can in principle compute it for an arbitrary numerical value of the invariant $s = (2m_t + E)^2$ but it is a rather complicated function of the ratio s/m_t^2. It is difficult to obtain such a function analytically (numerical study was done in [28]). However, the analytical result for the diagram in the threshold limit simplifies and leads to a logarithmic singularity in energy E (for the moment we forget about Coulomb singularities)

$$\alpha_s^2 \ln(E/m_t)\,, \quad E \to 0\,. \tag{12}$$

Within the effective theory approach this singularity splits between GF (the low energy singularity $\ln(E/\mu_f)$) and the high energy coefficient $C_h(\alpha_s)$ (the mass singularity $\ln(\mu_f/m_t)$)

$$\alpha_s^2 \ln(E/m_t) = \alpha_s^2 \left(\ln(E/\mu_f) + \ln(\mu_f/m_t)\right)\,. \tag{13}$$

In both limiting cases ($E = 0$, or calculation at threshold, and calculation within low-energy theory for the GF) the resulting integrals are simpler than the original integral for the diagram and can be done analytically. For one massive particle the calculation of diagrams is technically simpler and the complete expressions in full theory are available for some physical quantities [29]. The corresponding effective theory near the static limit is HQET which is also simpler than NRQCD. In this case one can analyze the cancellation of factorization scale dependence between matching (high energy) coefficients [30] and the low energy (HQET) amplitudes explicitly.

Collecting the above ingredients together allows one to have NNLO accuracy for $\Pi(E)$ near the threshold.

4. Solution for the low energy part.

The main dynamical object used to describe the $t\bar{t}$ system near the threshold is the non-relativistic Green's function $G = (H - E)^{-1}$. The Hamiltonian is represented in the form [31, 32, 33]

$$H = H_C + \Delta H, \quad H_C = \frac{p^2}{m} - C_F \frac{\alpha_s}{r} \tag{14}$$

with

$$\Delta H = \Delta V_{pot} - \frac{H_C^2}{4m_t} - \frac{3C_F\alpha_s}{4m}\left[H_0, \frac{1}{r}\right]_+ - \frac{4\pi\alpha_s}{m_t^2}\left(\frac{C_F}{3} + \frac{C_A}{2}\right)\delta(\vec{r})\,. \tag{15}$$

Constructing the Green's function is straightforward and can be done analytically within perturbation theory near Coulomb Green's function $G_C(E)$ or numerically (for complex values of E only that can be used to describe the production of particles with nonzero width) [34, 35, 36, 37, 38, 39, 40, 41, 42]. Results are presented basically as an expansion in consecutive orders

$$G = G_0 + \Delta G_1 + \Delta G_2 \tag{16}$$

to check the convergence of the approximations. Here:

- LO: Coulomb approximation, $G_0 = G_C$
- NLO: $\Delta G_1 \to O(\alpha_s)$ corrections from the static potential $V_{pot}(r)$
- NNLO: $\Delta G_2 \to O(\alpha_s^2)$ corrections from α_s^2 terms in the static potential $V_{pot}(r)$ and from the second iteration of the $O(\alpha_s)$ term in $V_{pot}(r)$, relativistic v^2 corrections.

The relativistic H_C^2 and anticommutator corrections in eq. (15) can be taken into account by the shift of the parameters of the Coulomb Green's function $E \to E + E^2/4m_t$ and $\alpha_s \to \alpha_s(1 + 3E/2m_t)$. In this respect the modified Coulomb approximation can be used as the leading order approximation. The analytical solution for Green's function is perturbative

$$G = G_C - G_C\Delta H G_C + G_C\Delta H G_C \Delta H G_C - \ldots \tag{17}$$

The $\overline{\rm MS}$-scheme for the static potential $V_{pot}(r)$ has been used in the solution. The numerical results obtained by the different authors are rather close to each other [43].

5. Features of the physical result.

The top quark width Γ_t plays a crucial role in the calculation of the $t\bar{t}$ production cross section near the threshold [8]. At the calculational level the width can be taken into account by a shift of the energy variable E. Operationally one can proceed as follows. The mass operator of the top quark is approximated by the expression $M = m_t - i\Gamma_t/2$. Then the kinematical variable $s - 4m_t^2$ relevant to the near-threshold dynamics is substituted with $s - 4M^2$ ($\sqrt{s} = E + 2m_t$) and one finds

$$s - 4M^2 = 4m_t(E + i\Gamma_t) + E^2 + \Gamma_t^2 \,.$$

Neglecting higher orders in E and Γ_t one obtains a recipe for taking into account the width Γ_t by the shift $E \to E + i\Gamma_t$. The dispersion relation for the vacuum polarization function $\Pi(E)$ has the form

$$\Pi(E) = \int \frac{\rho(E')dE'}{E' - E} \,.$$

With the shift recipe one finds

$$\sigma(E) \sim \text{Im}\ \Pi(E + i\Gamma_t) = \text{Im}\ \int \frac{\rho(E')dE'}{E' - E - i\Gamma_t} = \Gamma_t \int \frac{\rho(E')dE'}{(E' - E)^2 + \Gamma_t^2} \,. \qquad (18)$$

Because the point $E + i\Gamma_t$ lies sufficiently far from the positive semiaxis (and the origin) in the complex energy plane the cross section eq. (18) is calculable point-wise in energy. For the $b\bar{b}$ system where Γ_b is small the situation is different and only moments of different kinds, for instance,

$$\int_{4m_b^2}^{\infty} \frac{\rho(s)ds}{s^n}$$

are meaningful in the near-threshold Coulomb PT calculations. The hadronic cross section $\sigma(E)$ was obtained by many authors (as a review see, [43]). The normalized cross sections $R^v(E)$ for typical values $m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $\alpha_s(M_Z) = 0.118$ are plotted in Fig.1

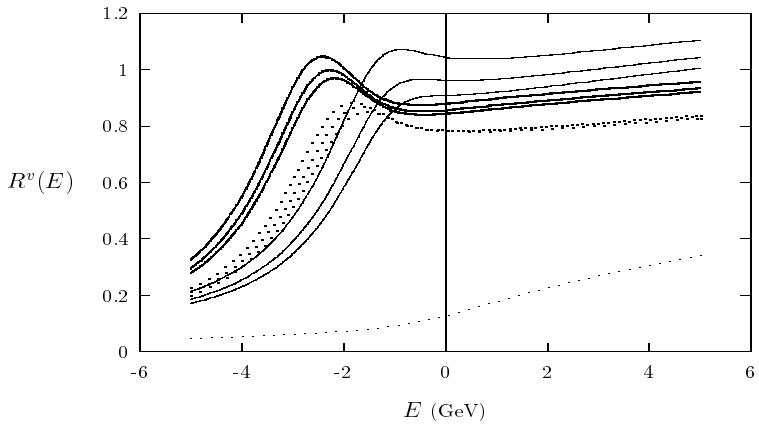

Fig. 1. The normalized cross section $R^v(E)$ in the leading order (solid lines), NLO (bold dotted lines) and NNLO (bold solid lines) for $m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $\alpha_s(M_Z) = 0.118$ and $\mu_s = 50$ GeV, 75 GeV and 100 GeV. The dotted line corresponds to the result in Born approximation.

[40]. The curves have characteristic points which are usually considered as basic observables. They are: E_p – the position of the peak in the cross section and H_p – the height of the peak in the cross section. In the limit of the small Γ_t (at least for Γ_t which is smaller than the spacing between the first two resonances) one would have $E_p \sim E_0$ and $H_p \sim |\psi_0(0)|^2$. For the actual value of $\Gamma_t = 1.43$ GeV which is larger than the spacing being the first two resonances the peak position and height are not determined by the first resonance only. The convergence for E_p and H_p in consecutive orders of perturbation theory near the Coulomb solution is not fast in the $\overline{\rm MS}$-scheme. For the typical numerical values of the theoretical parameters m_t, Γ_t and $\alpha_s(M_Z)$ one finds [40]

$$\begin{aligned} E_p &= E_0(1 + 0.58 + 0.38 + \ldots) \\ H_p &= H_0(1 - 0.15 + 0.12 + \ldots) \end{aligned} \tag{19}$$

(see also [44, 45]). Important contributions that affect the quality of convergence are the

local term ($\sim \alpha_s V_0 \delta(\vec{r})$ which is related to $1/r^2$ non-Abelian term [46]) and higher order PT corrections to $V_{pot}(r)$.

6. Possible improvements.

The slow convergence for the peak characteristics of the cross section has caused some discussion. The suggestions of the redefinition of the top quark mass have been made (as a review see [43], also [47, 48]). The use of the pole mass as a theoretical parameter for the description of the cross section near the threshold is criticized on the ground of its infrared instability [49]. It is usual that some effects of interaction can be taken into account by introducing the effective mass parameter for the particle [50, 51, 52]. In this talk I only discuss some possible ways of optimizing the convergence for the Green's function with the pole mass as a theoretical parameter. Note that actual calculations near the threshold have been performed within the pole mass scheme. For optimizing the convergence one can use methods of exact summation of some contributions in all orders and renormalization scheme invariance of PT series e.g. [34]. The Hamiltonian can be written in the form

$$H = H_{LO} + \Delta V_{PT} + \alpha_s V_0 \delta(\vec{r}), \quad V_0 = -\frac{4\pi}{m_t^2}\left(\frac{C_F}{3} + \frac{C_A}{2}\right) \tag{20}$$

where corrections are given by the perturbation theory corrections to $V_{pot}(r)$ (ΔV_{PT}-part) and by the local term ($\alpha_s V_0 \delta(\vec{r})$-part). These two contributions can be dealt with more accurately than in the straightforward approach. Indeed, the $\delta(\vec{r})$-part is a separable potential and can be taken into account exactly [53]. The solution reads

$$G(E;0,0) = \frac{G_{ir}(E;0,0)}{1 + \alpha_s V_0 G_{ir}(E;0,0)} \tag{21}$$

with

$$G_{ir}(E) = (H_{LO} + \Delta V_{PT} - E)^{-1} \tag{22}$$

being the irreducible Green's function. Dealing with the PT expansion of the static potential in NRQCD is an important issue in getting stable results for the cross section near the threshold because the static potential is the genuine quantity which is computed in high

order of PT in the strong coupling constant [54]. The convergence in the $\overline{\rm MS}$ scheme is not fast which reflects the physical situation that the observables represented by the cross section curve (for instance, E_p and H_p) are sensitive to different scales. The finite-order perturbation theory expansion of the static potential given in eq. (6) cannot handle several distinct scales with the same accuracy. Indeed, the PT expansion of the static potential is done near some (arbitrary) scale (or distance) which can be considered simply as a normalization point. The farther a given point lies from this normalization point the worse the precision of the PT expansion for the static potential at this point is. The PT expressions in the $\overline{\rm MS}$ scheme are not directly sensitive to physical scales because subtractions are made in a mass independent way (for instance, massive particles do not decouple automatically in the $\overline{\rm MS}$ scheme e.g. [55]). Therefore it is instructive to rewrite the static potential in more physical terms than just the $\overline{\rm MS}$-scheme parameters $(\alpha_{\overline{\rm MS}}, \mu)$ (which would remind the reader of the momentum subtraction scheme where, for instance, the decoupling is explicit [56])

$$V_{pot}(r) \;=\; -C_F \frac{\alpha_0}{r}\left(1+\alpha_0 b_1 \ln r/r_0 + \alpha_0^2(b_1^2 \ln^2 r/r_0 + b_2' \ln r/r_0 + c) + \ldots\right) . \qquad (23)$$

Here r_0 and c are the parameters of the renormalization scheme freedom in NNLO and α_0 is the corresponding coupling in the $\{r_0, c\}$-scheme [57]. They parameterize the center of the expansion (a normalization point) and the derivative (respective β-function) of the static potential. The parameters (r_0, c) can be chosen such in order to minimize the higher order corrections to a particular observable (e.g. [34] where NLO analysis has been done). In such a case r_0 can be understood as a typical distance to which a chosen observable is sensitive. Note that the best approximation of the static potential $V_{pot}(r)$ for different scales would be provided by the use of the running coupling constant $\alpha_s(r)$. The analytical calculation of the Green's function becomes just impossible in this case. However, even for the numerical calculation of the Green's function one cannot naively use the running coupling constant $\alpha_s(r)$ in the static potential $V_{pot}(r)$ for all r. With a generic running coupling constant, the IR singularity (Landau pole) can occur in $\alpha_s(r)$ at large r which are formally necessary

for computing the Green's function. This singularity has little to do with usual divergences within the effective theory approach and cannot be removed by the standard renormalization tools. It can be dealt with if an IR fixed point appears in the evolution for the effective coupling constant (e.g. [58]) or with some other regularization e.g. [59]. For the top quark production the contribution of this area (large r) into the cross section is small because of the large decay width of the top quark. In the finite-order PT analysis the parameters r_0 and c can be chosen to minimize higher order corrections either to E_p or to H_p but not to both simultaneously because E_p and H_p are sensitive to different distances. Indeed, one finds the difference of scales minimizing corrections to the first Coulomb resonance in NLO to be

$$\ln(r_E/r_\psi) = \frac{1}{3} + \frac{\pi^2}{9}\,. \tag{24}$$

Because of the large top quark width many states (resonances and continuum alike) contribute into the position and height of the peak in the cross section. Therefore the characteristic distance estimates are not so transparent (the NNLO peak position, for instance, is not exactly the ground state energy in zero width limit). The relation (24) can serve just as a basic guide. In practical analysis one can choose the particular numerical values for the parameters (r_0, c) which stabilize either E_p or H_p.

7. Conclusion

To conclude, we have presented a cross section for the $t\bar{t}$ pair production near the threshold. The result is based on the solution to the Schrödinger equation for the nonrelativistic Green's function. Some methods of resummation and convergence optimization are discussed.

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